

An introduction to the modelization of multiphase flows

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Outline

- 1 Me, myself and I
- 2 Modelizing fluid mechanics: the Navier-Stokes equations
- 3 About my work

Goal of the talk

*Derivation and analysis of models
for compressible multiphase flows*

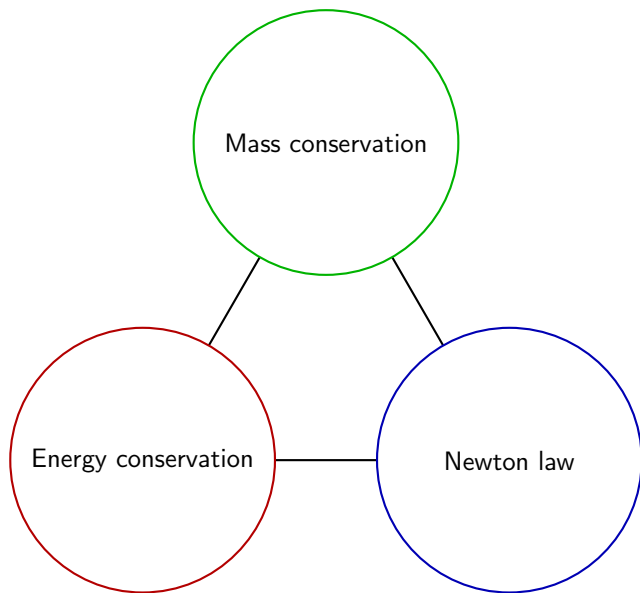
Modelization

- **Fluids:** compressible, incompressible. One or several fluids (mixture).
Sediments.
- **Variables:** density, velocity, pressure, temperature, internal energy ...
- **Phenomena:** exchange of mass, viscosity effects, friction, phase change ...
- **Scales:** microscopic (fluid particles) to huge (ocean currents, atmosphere)

The Navier-Stokes equations (raw)

$$\begin{aligned}\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) &= \mathbb{M}, \\ \partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p &= \mathbb{S}, \\ \partial_t (\rho E) + \nabla \cdot ((\rho E + p) \mathbf{u}) &= \mathbb{T}.\end{aligned}$$

The Navier-Stokes equations



Then what ?

- ➊ Derivation of alternate models to match the precise phenomenon we study.
- ➋ Adapting the boundary conditions and the geometry of the domain.
- ➌ Numerical simulations.
- ➍ Asymptotic studies.
- ➎ Well-posedness of models.
- ➏ Solving the equations.

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Derivation and analysis of models for compressible multiphase flows

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 - Averaged models
 - First work
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Examples of multiphase flows

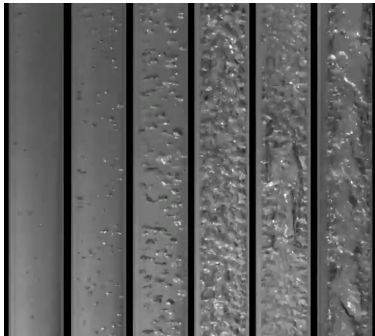


Figure: Different regimes of bubbly flows.

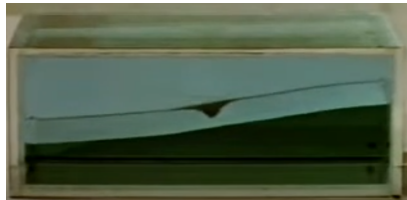


Figure: Stratified flow.

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About averaging

Why?

- Navier-Stokes mathematically unsolvable.
- Numerical simulation too expensive.
- Navier-Stokes is too precise.¹

¹Drew and Passman. *Theory of Multicomponent Fluids*. Springer New York, 1999

²Ishii and Hibiki. *Thermo-Fluid Dynamics of Two-Phase Flow*. Springer, 2011

About averaging

Why?

- Navier-Stokes mathematically unsolvable.
- Numerical simulation too expensive.
- Navier-Stokes is too precise.¹

How?

- Apply averaging operators.
- Different types of averaging: time, space.²
- Homogenization.

¹Drew and Passman. *Theory of Multicomponent Fluids*. Springer New York, 1999

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Challenges

- Differentiate velocities, pressures, temperatures...

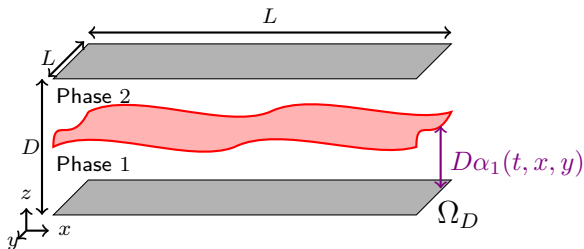
Challenges

- Differentiate velocities, pressures, temperatures...
- Well-posedness: in what sense ? During how much time ?

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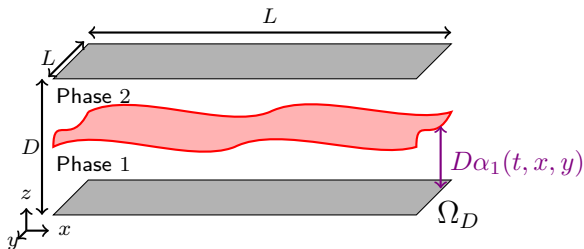
Flow configuration



Assumptions

- Immiscible fluids, stratification preserved through time.
- Barotropy: the pressure only depends on the density $\Rightarrow$ no energy equation.

Flow configuration



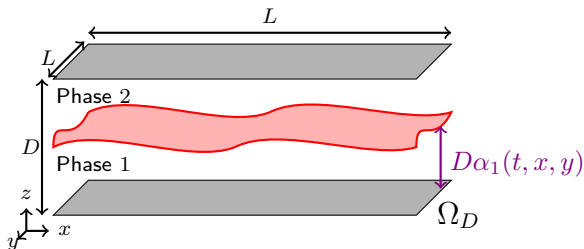
Assumptions

- Immiscible fluids, stratification preserved through time.
- Barotropy: the pressure only depends on the density $\Rightarrow$ no energy equation.

Goal

Obtain an averaged model with two velocities and two pressures (Baer-Nunziato).

Flow configuration



- Viscous fluid.
- Friction at the boundary and at the interface.
- Continuity of the stress at the interface.

Assumptions

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- **Consequence:** the velocities and the densities don't vary much in the vertical direction.

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Hope

Finding a model written in terms of vertical averages might be relevant.

Derivation protocol

- 1 Rescale the equations, boundary conditions and interface conditions.

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- 3 Integrate the two Navier-Stokes systems in the vertical direction.

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- 1 Rescale the equations, boundary conditions and interface conditions.
- 2 Neglect terms of high order in ε .
- 3 Integrate the two Navier-Stokes systems in the vertical direction.
- 4 Close the remaining boundary terms using the boundary conditions and estimates.

Conclusion

Our protocol enables us to derive:

- ① Two-velocity, one-pressure models (ill-posed).
- ② One-velocity, one-pressure models.
- ③ Two-velocity, one-pressure and two-temperature models in the Navier-Stokes Fourier case.
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Perspectives

- Study the well-posedness of the microscopic model.
- Numerical simulation of the one-velocity, one-pressure mixture model.
- Work on a multilayer two-phase flow model.

Thank you for your attention!

References

- [DP99] Drew and Passman. *Theory of Multicomponent Fluids*. Springer New York, 1999.
- [IH11] Ishii and Hibiki. *Thermo-Fluid Dynamics of Two-Phase Flow*. Springer, 2011.